Homework Solution "Work and Energy"

1) Problem: Let F = k/x^2. What is the potential energy function of this force?

Solution: $\Delta PE = W = \int F dx = \int_{x_o}^{x_f} \frac{k}{x^2} dx = -\frac{k}{x}$

2) Problem: A certain spring does not follow Hooke's law but rather

$$F(x) = -80x - 15x^2$$

A 2.0 kg mass is attached to this spring, pulled a distance 1.0 m on a frictionless surface and released. What is the object's velocity when it passes through the equilibrium (x=0) point?

Solution: Find the work by integrating over the displacement, then using conservation of Energy find the velocity.

$$W = \int F dx = \int \left(-80x - 15x^2\right) dx = \int \left(-80x\right) dx + \int \left(-15x^2\right) dx$$

$$= -\frac{80}{2} x^2 - \frac{15}{3} x^3 = -40x^2 - 5x^3$$

The force is negative because it is a restoring force (spring) so multiply a negative sign through giving

$$40x^2 + 5x^3$$

Now

$W = \Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2$ and $v_o = 0$ so $40x^2 + 5x^3 = \frac{1}{2} m v_f^2$

Solving for v_f :

$$v_f = \sqrt{\frac{2\left(40x^2 + 5x^3\right)}{m}} = \sqrt{\frac{2\left(40(1m)^2 + 5(1m)^3\right)}{2kg}} = 6.71 \frac{m}{s}$$