Homework Solution "Fluids"

Problem: What is the hydrostatic force at the back of Grand Coulee Damn if it has a height of 150 m and a width of 1200 m. [Note: hydrostatic means 'due to the water alone', that is we neglect the external pressure of the atmosphere.]

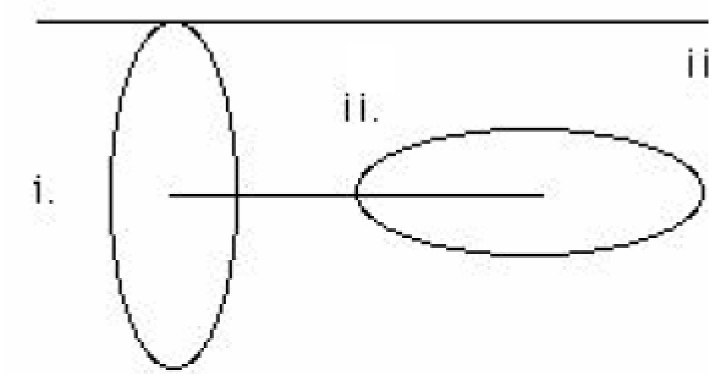
Solution: $F_{\textbf{rectangle}} = \dfrac{\rho g w h^2}{2} = \dfrac{(1000)(9.8)(1200)(150)^2}{2} = 1.3\times10^{11}\,N$

Problem: Why are dam's thicker at the bottom than at the top?

Solution: To keep its pants up… :-)

No, its because the pressure is grater at the bottom.

Problem: An ellipse has a major axis a and a minor axis b. What would be the pressure of a fluid with density on an ellipse if i) the major axis was vertical, ii) the major axis was horizontal

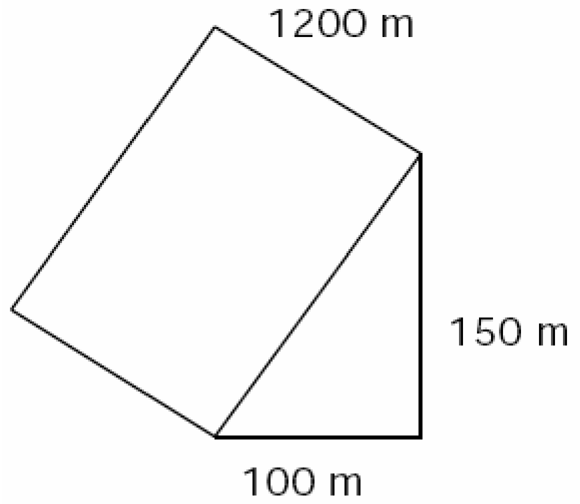


Solution: The area of an ellipse is $A = \pi ab$ and $F = P_{ave}A$ so:

i) $P_{ave}A = \left(\dfrac{\rho g a}{2}\right)(\pi ab) = \dfrac{\rho \pi g a^2 b}{2}$

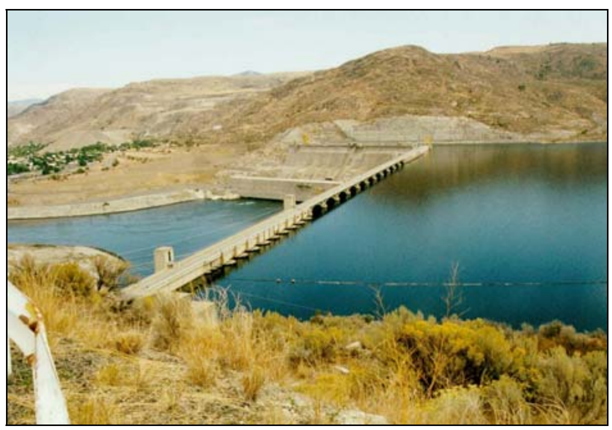
ii) This has the same answer as i.

Problem: Grand Coulee Damn is actually 100 m thick at the bottom and 0 thick at the top. What is the total force on the dam (not a plug in since the triangle is inverted).



Solution: First off, this problem is designed to make you think. What are the forces on a dam? Well there is a hydrostatic force on the 'back' of the dam from the water it holds back. Then there is air on the 'front' side of the dam and usually rock on the sides where the dam is anchored. The Grand Coulee Dam is on the Columbia river and is anchored this way.

Front



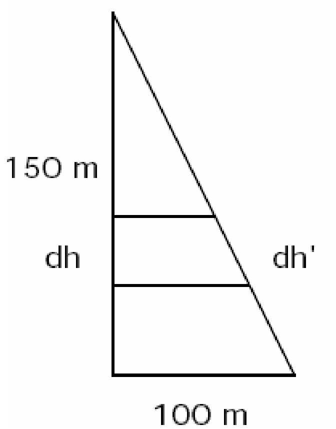
Back

Photos courtesy of: http://users.owt.com/chubbard/gcdam/html/photos/exteriors.html

We found the hydrostatic force on the back side of the dam (problem #1) to be $1.3x10^{11}$N. We can find the force from the atmosphere on the front of the dam:

$$F = \int PdA = \int \rho gwhdh'$$

Where:
ρ is the density of air (1.3 kg/m^3 - from CRC Manual)
w is constant at 1200m
h' is the hypotenuse



dh'/dh = the hypotenuse / the height

hypotenuse = $\sqrt{100^2+150^2}$ = 180

so, dh'/dh = 180 / 150 = 6/5

which means that dh' = 6/5 dh

$$F=\int PdA=\int \rho gwhdh'=\rho gw\int h\frac{6}{5}dh=\frac{6}{5}\rho gw\int hdh=\frac{6}{5}\rho gw\frac{h^2}{2}$$
$$=\left(\frac{6}{5}\right)(1.3)(9.8)(1200)\left(\frac{150^2}{2}\right)=2.06\times10^8\,N$$

As you can see, the force from the atmosphere on the 'front' of the dam is a minor force compared to the force of the water on the 'back', three orders of magnitude!

As for the sides, we do not know (at least I do not) how the anchor is designed. Is the rock pushing in compressing the dam, or is the dam flexing out pushing against the rock? I think that the later makes more sense, as this would mean that the surrounding rock would then take up some of the force load and that the anchor would become more secure as you put a greater load on it (more water behind the dam).

Problem: Show that Reynolds Number is dimensionless.

Solution:

$$R_n=\frac{\rho vL}{\eta}$$
$$\rho=\frac{M}{L^3}$$
$$v=\frac{L}{T}$$
$$\eta=\frac{M}{LT}$$
$$R_n=\frac{\left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)L}{\left(\frac{M}{LT}\right)}=\left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)L\left(\frac{LT}{M}\right)$$

These all cancel, so it is dimensionless.