Homework Solution "Gravity"

Problem: Look up values of M_e and R_e and show that G M_e/R_e^2 = 9.8 m/s^2

Solution:

$$g = G\frac{M_e}{R_e^2} = \left(6.67\times10^{-11}\right)\frac{\left(5.97\times10^{24}\right)}{\left(6.38\times10^{6}\right)^2} = 9.783$$

Problem: Find the integral for the potential energy and show that when $r_o = R_e$ and $r_1 = R_e + h$, U = mgh when h is small compared to R_e.

Solution:

$$PE = \int F(x)\,dx$$

$$\therefore$$

$$PE_g = \int_{R_e}^{R_e+h} F(r)dr = \int_{R_e}^{R_e+h} G\frac{M_e m}{r^2}dr = -G\frac{M_e m}{r}\Bigg]_{R_e}^{R_e+h}$$

$$PE_g = -G\frac{M_e m}{R_e+h} + G\frac{M_e m}{R_e} = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e+h}\right)$$

$$PE_g = GM_e m\left(\frac{R_e - R_e + h}{R_e^{\,2} + R_e h}\right) = GM_e m\left(\frac{h}{R_e^{\,2} + R_e h}\right)$$

$$PE_g = GM_e m\left(\frac{h}{R_e^{\,2} + R_e h}\right)\left(\frac{R_e}{R_e}\right) = GM_e m\left(\frac{R_e h}{R_e^{\,3} + R_e^{\,2} h}\right)$$

$$PE_g = GM_e m\left(\frac{h}{R_e^{\,2}}\right)\left(\frac{R_e}{R_e+h}\right) = G\frac{M_e mh}{R_e^{\,2}}\left(\frac{R_e}{R_e+h}\right)$$

because $G\frac{M_e m}{R_e} = g$

$$PE_g = gmh\left(\frac{R_e}{R_e+h}\right)$$

and h is negligable compared to R_e

$$PE_g = gmh(1) = gmh \quad \text{Q.E.D.}$$