1. Suppose the density of a rod 1 m long with one end at the origin of the x-axis was given as $\rho(x) = 3 - x^2$. Where is its center of mass.

$$x_{cm} = \int \rho(x_i, y_i)\, x \, dx / \int \rho(x,y)\, dx$$

$$x_{cm} = \int (3 - x^2) x \, dx / \int (3 - x^2) dx$$
$$= [\int 3x dx - x^3 dx] / [\int 3 dx - \int x^2 dx]$$
$$= [3/2 x^2 - x^4/4] / [3x - x^3/3]^1_0$$
$$= [3/2 - 1/4] / [3 - 1/3]$$
$$= [5/4] / [8/3]$$
$$= 15/32 = 0.469 \text{ m}$$

2. Suppose the rod in 1 had uniform density $\rho = M/L$. Where is its center of mass?

A real toughie.

$$x_{cm} = \int \rho(x_i, y_i)\, x \, dx / \int \rho(x,y)\, dx$$

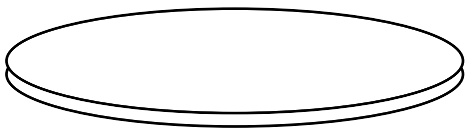
$$x_{cm} = \int [M/L] x \, dx / \int [M/L] dx$$
$$= [[M/L] \int x dx / [M/L] [\int dx]$$
$$= \int x dx / [\int dx]^1_0$$
$$= [x^2/2] / x|^1_0$$
$$= x/2|^1_0$$
$$= 0.5 \text{ m}$$

3. Calculate the moment of inertia for the rod of length L of uniform density $\rho = M/L$ about one of its ends. Compare your result with what we got from the parallel axis theorem in class.

$$I = \int [M/L] x^2 \, dx$$
$$= [M/L] \int x^2 \, dx |^L_0$$
$$= [M/L]\, L^3/3 = ML^2/3$$

which is the same as before

4. Calculate the moment of inertia of a) a ring and b) a solid disk for rotation about a point on its edge



$I = I_{cm} + md^2$

where d is the distance between the center of mass and the axis you wish to evaluate the moment of inertia.

In this case, d = r, the radius

ring

$I_{cm,\ ring} = mr^2$

$I_{ring} = mr^2 + mr^2 = \mathbf{2\ mr^2}$

disk

$I_{cm,\ disk} = 1/2\ mr^2$

$I_{ring} = 1/2\ mr^2 + mr^2 = \mathbf{3/2\ mr^2}$