

## Homework

1) A car of 110 kg is supported by 4 vertical springs. When a man pushes down on the car and then lets go, the car rocks up and down with a period of 0.75 s. What are the frequency, the angular frequency and the spring constant of each spring. Assume the springs are identical and that the shock absorbers are completely shot so that the motion is SHM.

$$f = 1/T = 1/0.75 = 1.33 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi (1.33) = 8.37 \text{ rad/s}$$

since each spring supports 1/4 of the mass, 27.5 kg

$$\omega = [k/m]^{1/2} \implies k = m \omega^2 = (27.5)(8.37)^2 = 1.9 \times 10^3 \text{ N/m}$$

2) Deuterium, D, is an isotope of hydrogen with a mass of 1.998 times that of regular hydrogen. If the regular hydrogen molecule, H<sub>2</sub>, vibrates at  $1.31 \times 10^{14}$  Hz and the 'spring' connecting the H<sub>2</sub> and the D<sub>2</sub> is the same, at what frequency will a D<sub>2</sub> molecule vibrate?

since the 'springs' are the same, k's are the same

$$k_D = k_H$$

$$m_D \omega_D^2 = m_H \omega_H^2$$

$$\begin{aligned} \omega_D &= \omega_H [m_H/m_D]^{1/2} \\ &= 1.31 \times 10^{14} [1/1.998]^{1/2} = 9.3 \times 10^{13} \text{ Hz} \end{aligned}$$

3) The mass of astronauts cannot be determined with a scale in space because they are 'weightless', that is, in free fall. So what they do to determine mass in space is to sit in a special vibrating chair and measure the frequency of vibration.

On earth a mass of 66.91 kg was placed in the chair, which was found to oscillate with a period of 2.088 s.

In space an astronaut sits in the chair and is oscillates with a period of 2.299 s. What is the mass of the astronaut?

It's the same chair, so the k's are the same

$$m \omega^2 = m' \omega'^2$$

primed quantities are in space

$$\omega = 2\pi f$$

$$T = 1/f$$

so

$$\omega = 2\pi/T$$

$$m \omega^2 = m' \omega'^2 \text{ becomes}$$

$$m(2\pi/T)^2 = m'(2\pi/T')^2$$

$$m' = m (T'/T)^2 = 66.91 (2.299/2.088)^2 = 81.1 \text{ kg}$$

4) A mass of 8.0 kg is attached to a spring, which oscillates with an amplitude of 0.25 m and a frequency of 0.60 Hz. What is the total energy of the system?

$$K = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t)$$

$E = K + U$ ; K largest when  $\sin = 1$ , which is when  $U = 0$

$$K_{\max} = \frac{1}{2} m A^2 \omega^2 = \frac{1}{2} m A^2 (2\pi f)^2 = \frac{1}{2} (8)(.25)^2 (2\pi \cdot 0.60)^2 = 3.5 \text{ J}$$

5) Where in its motion is the kinetic energy of a simple harmonic

oscillator at a minimum? At a maximum?

K minimum when  $x = A$  (at ends, fully displaced), K max when  $x = 0$  (in center)

Where in its motion is the potential energy of a simple harmonic oscillator at a minimum? At a maximum?

U is the opposite of K