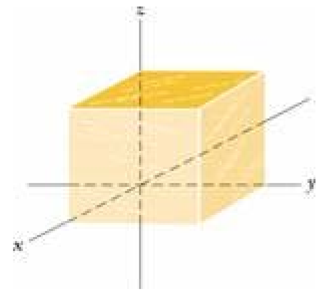
***Homework Solution "Gauss' Law"***

**Problem:**

A Cube with sides of length A, is in an uniform electric field $\left|\vec{E}\right|$ that points in the $+\hat{X}$ direction.




**a)** Find the flux through *each* of the six sides.
**b)** Find the total flux through the cube.
Be sure to show all of your work. **Hint:** Set the coordinate system so that the back left corner is at the origin as in the diagram.

**Solution:**

The flux through a side is described by $\Phi = \int \vec{E} \bullet d\vec{A}$. We can first deal with the dot product by reducing the argument of the integral to $\int ECos\theta dA = ECos\theta \int dA$ and then moving the Electric field term and Cosine out of the integral, as they are both constants. Each side of the cube has length A and the area of any side is simply A². Because we are integrating over the area of sides of each side, the limits of integration are 0 to A². This reduce the flux to a general solution of $\Phi = EA^2 Cos\theta$.

**a)** Recall that Theta ($\theta$) is the angle between the area and Electric field. Four of the six sides of the cube have a value of Theta that is $90°$. The sides are the top and bottom of the cube, which are both in the *X*-*Y* plane, and the two sides that are in the *X*-*Z* plane. For these four sides the Cosine function goes to zero, so $\Phi_1 = EA^2Cos(90°) = 0$, $\Phi_2 = EA^2Cos(90°) = 0$, $\Phi_3 = EA^2Cos(90°) = 0$, $\Phi_4 = EA^2Cos(90°) = 0$. This means that there is no flux through these sides, which makes physical sense if you consider that the Electric field is in the $+\hat{X}$ direction so it goes past, but not through these four sides.

The last two sides, let's call them the front and back, do have the Electric field passing through them so they should have a non-zero value for the flux. The back side has its corner at the origin and is in the *Y*-*Z* plane. The normal of the area of the back side points in the $-\hat{X}$ direction, so the angle between the area and Electric field for this side has a value of Theta that is $180°$. For the back side then the Cosine function goes to -1 , so $\Phi_5 = EA^2Cos(180°) = EA^2(-1) = -EA^2$.

The front side is also in the *Y*-*Z* plane. The normal of the area of the front side points in the $+\hat{X}$ direction, area and Electric field point in the same direction for this side. The angle between them then has a value of Theta that is $0°$. For the font side then the Cosine function goes to 1 , so $\Phi_6 = EA^2Cos(0°) = EA^2(1) = EA^2$.

**b)** To find the total flux through the cube we simply sum all of the individual fluxes.

$$\Phi_{net} = \sum_{i=1}^{6} \Phi_i = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = 0 + 0 + 0 + 0 + EA^2 + \left(-EA^2\right) = 0$$

So the net flux is zero, which makes physical sense if you consider that all of the Electric field that went into the cube also left the cube.