Homework on Potential - Answers

1. An stationary electron is infinitely far from a proton. (remember the potential inifinitely far from a point charge is 0). What is its velocity when it falls to 5.3 x $10^{-11}$ m?

Use conservation of energy

$E_i = E_f$

$K_i + U_i = K_f + U_f$

$0 + 0 = 1/2\, m_e v_f^{\,2} + q_e V_f$ [inital U = 0 by definition]

$V_f = [1/4\pi e_o]\, q_p / r_f$

subscripts e and p refer to electron and proton

$v_f = \{[-2/4\pi m_e e_o]\, q_p / r_f\}^{1/2}$

putting in values (don't forget sign of electron's charge!), we get

*$v_f = 3.1 \times 10^6$ m/s*

2. Calculate the electric field of a point charge q' from the electric potential of the point charge.

We need to take the gradient of the potential

$$\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} = -E$$

or, for r

$$E(r) = -\frac{dV}{dr}\hat{r}$$

from the lecture

$$V(r) = \frac{q}{4\pi\varepsilon_o r}$$

so

$$E(r) = \frac{q}{4\pi\varepsilon_o r^2}$$

3. The potential at a distance z along the axis of a ring of radius R that has a charge Q uniformly distributed along its circumference is

$$V(z) = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{(z^2 + R^2)^{1/2}}\right]$$

Calculate the z component of the electric field.

Same idea as previous problem, that is, take the derivative of V with respect to z (that is, get the gradient)

$$E(z) = -\frac{dV(z)}{dz}$$

AAACK! How do we take the derivative of this horror. We could copy it from the homework of someone who knows what they are doing, but it is better (test wise) if we actually know how to do it. We use the CHAIN RULE

$$\frac{df}{dx} = \frac{df}{dg} \quad \frac{dg}{dx}$$

here

$$g(z) = (z^2 + R^2)$$

and

$$f(z) = \frac{Q}{4\pi\varepsilon_o} \left[ \frac{1}{(g(z))^{1/2}} \right]$$

and the independent variable z is what we are differentiating with respect to (rather than x)

Using the usual derivative rules

$$\frac{df}{dg} = \frac{1}{4\pi\varepsilon_o} \quad -1 \frac{1}{2(z^2 + R^2)^{3/2}}$$

$$\frac{dg}{dz} = 2z$$

which gives us

$$E(z) = -\frac{df}{dz} = -1 \frac{1}{4\pi\varepsilon_o} \quad -1 \frac{1}{2(z^2 + R^2)^{3/2}} * 2z$$

upon simplification

$$E(z) = \frac{1}{4\pi\varepsilon_o} \frac{z}{(z^2 + R^2)^{3/2}}$$

Why don't we have to calculate the x and y components?

They cancel - for every contribution +E(y) on one side of the distribution, there is a contribution of -E(y) on the opposite side.