Homework Solution “Waves”

A) In order to show that $y = Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L}$ is a solution of the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ we first take the second partial derivative of *y* with respect to *x:*

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2\left(Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L}\right)}{\partial x^2} = -\frac{\pi^2}{L^2}Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L}$$

We next take the second partial derivative of *y* with respect to *t:*

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2\left(Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L}\right)}{\partial t^2} = -\frac{\pi^2 c^2}{L^2}Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L}$$

Now we substitute these solutions into the wave equation and see if it is satisfied:

$$-\frac{\pi^2}{L^2}Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L} = -\frac{1}{c^2}\frac{\pi^2 c^2}{L^2}Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L}$$

The $\frac{1}{c^2}$ and c^2 term cancel, so yes, $y = Sin\frac{\pi x}{L}Cos\frac{\pi ct}{L}$ is a solution of the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$!

B) At x=0 we get $-\frac{\pi^2}{L^2}Sin\frac{\pi 0}{L}Cos\frac{\pi ct}{L} = -\frac{1}{c^2}\frac{\pi^2 c^2}{L^2}Sin\frac{\pi 0}{L}Cos\frac{\pi ct}{L} = 0$ as $Sin 0 = 0$.

Likewise at x=L we get $-\frac{\pi^2}{L^2}Sin\frac{\pi L}{L}Cos\frac{\pi ct}{L} = -\frac{1}{c^2}\frac{\pi^2 c^2}{L^2}Sin\frac{\pi L}{L}Cos\frac{\pi ct}{L} = 0$ as $Sin\pi = 0$.

This describes the condition where a vibrating string is fixed at the ends.
What it does not describe is the condition where one end is not fixed, like a whip for example.

Yeehaw! Rawhide!